Suppose that $\vec{b}$ is a vector of magnitude 3, and $\vec{d}$ is a vector of magnitude 2, and the angle between $\vec{b}$ and $\vec{d}$ is $\frac{5\pi}{6}$ radians. Fill in the blanks.

SCORE: ____ / 3 PTS

[a] $\|\vec{b} \times \vec{d}\| =$ 3 (2) $= \|\vec{b}\| \|\vec{d}\| \sin\theta = 3 \cdot 2 \sin\frac{5\pi}{6}$

[b] $\vec{b} \times \vec{b} =$ $\vec{0}$ (1) ← MUST BE A VECTOR, NOT A NUMBER

Fill in the blanks. GRADED BY ME

SCORE: _____ / 6 PTS

NOTE: For each part (ie. [a], [b], [c]), you must fill in all blanks correctly to receive any credit.

[a] If plane $\wp_1$ is perpendicular to plane $\wp_2$, then the NORMAL vector of plane $\wp_1$ is PERPENDICULAR to the NORMAL vector of plane $\wp_2$.

[b] If line ℓ_1 is parallel to line ℓ_2, then the DIRECTION vector of line ℓ_1 is PARALLEL to the DIRECTION vector of line ℓ_2.

[c] If line ℓ is parallel to plane $\wp$, then the DIRECTION vector of line ℓ is PERPENDICULAR to the NORMAL vector of plane $\wp$.

Fill in the blanks. SCORE: _____ / 4 PTS

[a] If $\vec{u} \times \vec{v} = < -3, 2, -5 >$, then $\vec{v} \times \vec{u} =$ $\langle 3, -2, 5 \rangle$. (1) $= -\vec{u} \times \vec{v}$

[b] If $(\vec{u} \times \vec{v}) \cdot \vec{w} = -12$, then $\vec{u} \cdot (\vec{v} \times \vec{w}) =$ -12. (1) $= (\vec{u} \times \vec{v}) \cdot \vec{w}$

[c] If $\vec{u}, \vec{v}$ and $\vec{x}$ are adjacent edges of a parallelepiped, and $\vec{u} \times \vec{v} = < 3, -2, -5 >$ and $\vec{x} = < 1, -3, 3 >$,

then the volume of the parallelepiped is 6. (2) $= |(\vec{u} \times \vec{v}) \cdot \vec{w}| = |3 + 6 - 15|$

Let P be the point $(-5, -1, 3)$.
Let Q be the point $(-4, 1, 4)$.
Let R be the point such that $\overrightarrow{PR} = 3\vec{j} + 2\vec{k}$.

ALL ITEMS ① POINT UNLESS OTHERWISE NOTED

SCORE: _____ / 17 PTS

[a] Find parametric equations for the line which is parallel to $\frac{x+4}{6} = 3 - y = \frac{z-5}{2}$, and also contains Q.

$x = -4 + 6t$
$y = 1 - t$
$z = 4 + 2t$

$\frac{x+4}{6} = \frac{y-3}{-1} = \frac{z-5}{2}$ $\vec{d} = \langle 6, -1, 2 \rangle$

[b] Find the standard (point-normal) equation of the plane which is parallel to both $\overrightarrow{PQ}$ and $\overrightarrow{PR}$, and also contains P.

$\overrightarrow{PQ} = \langle 1, 2, 1 \rangle$

$\vec{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 1 \\ 0 & 3 & 2 \end{vmatrix} = \langle 1, -2, 3 \rangle$ ②

CHECK
$\vec{n} \cdot \overrightarrow{PQ} = 1 - 4 + 3 = 0$
$\vec{n} \cdot \overrightarrow{PR} = -6 + 6 = 0$ ✓

$(x+5) - 2(y+1) + 3(z-3) = 0$

[c] Find the angle between the plane in part [b] and the plane $3x + y + 2z = 7$.

$\cos^{-1} \frac{|\vec{n}_1 \cdot \vec{n}_2|}{\|\vec{n}_1\| \|\vec{n}_2\|} = \cos^{-1} \frac{|\langle 1, -2, 3 \rangle \cdot \langle 3, 1, 2 \rangle|}{\|\langle 1, -2, 3 \rangle\| \|\langle 3, 1, 2 \rangle\|} = \cos^{-1} \frac{|3 - 2 + 6|}{\sqrt{14}\sqrt{14}}$

$= \cos^{-1} \frac{7}{14} = \cos^{-1} \frac{1}{2} = \frac{\pi}{3}$

[d] S is a point such that $PQSR$ is a parallelogram. Find the area of parallelogram $PQSR$.

$\|\overrightarrow{PQ} \times \overrightarrow{PR}\| = \|\langle 1, -2, 3 \rangle\| = \sqrt{14}$

[e] Find a vector of magnitude 3 perpendicular to both $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

$\frac{3}{\|\overrightarrow{PQ} \times \overrightarrow{PR}\|} (\overrightarrow{PQ} \times \overrightarrow{PR})$ ② $= \frac{3}{\sqrt{14}} \langle 1, -2, 3 \rangle$